Week 7 Worksheet Tuesday

Instructions. Congratulations on conquering the midterm1 monster! It's time to move on to the next task! Discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Topic: Derivative Computing

Common Derivatives that we learned so far:

$(x^n)' =$ nx^{n-1} $\quad$ $\frac{d}{dx}[\sin x] =$ $\cos x$

$\frac{d\cos x}{dx} =$ $-\sin x$ $\quad$ $(\tan x)' =$ $\sec^2 x$

1. Compute the Derivatives of the following functions (No need to use definition.)

   (a) $y = 5x^3 + \sqrt{x}$

   (b) $f(x) = \sqrt{x^3}\sin x$

   (c) $f(x) = \cot x$ $\quad$ (Hint: $\cot x = \frac{\cos x}{\sin x}$)

   (d) $y = \frac{\tan x}{x^2+3}$

   (e) $y = \left(3x^2 - \sqrt{x} + 1\right)^5$

   (f) $y = \sqrt{1+x^3}$

   Chain rule:

   (g) $y = \sin(2x) - \cos(3x)$

   (h) $f(x) = (\cos x)^2 - \cos(x^2)$

   (i) $y = \sin(\cos(3x))$

   (j) $f(x) = (\sin x)^2(1+x^2)$

(a) $y = 5x^3 + x^{\frac{1}{2}}$

$y' = 15x^2 + \frac{1}{2}x^{-\frac{1}{2}}$

(b) $f(x) = x^{\frac{3}{2}}\sin x$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}\sin x + x^{\frac{3}{2}}\cos x$

(c) $f(x) = \frac{\cos x}{\sin x}$

$f'(x) = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= -\frac{1}{\sin^2 x}$

$= -\csc^2 x$

(d) $y' = \frac{\sec^2 x \cdot (x^2+3) - 2x\tan x}{(x^2+3)^2}$

(e) $y' = 5(3x^2 - \sqrt{x} + 1)^4(6x - \frac{1}{2}x^{-\frac{1}{2}})$

(f) $y = (1+x^3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(1+x^3)^{-\frac{1}{2}} \cdot 3x^2$

(g) $y' = 2\cos(2x) + 3\sin(3x)$

(h) $f'(x) = 2\cos x \cdot (-\sin x) + \sin(x^2) \cdot 2x$

(i) $y' = \cos(\cos(3x)) \cdot (-\sin(3x)) \cdot 3$

(j) $f'(x) = 2\sin x \cos x (1+x^2) + (\sin x)^2 \cdot 2x$

chain rule ← (k*) $f(x) = \cos(x^2)\tan(\sqrt{x+1})$ *(2013fall exam)*

$f(x) = \cos(x^2)\tan((x+1)^{\frac{1}{2}})$

$f'(x) = -\sin(x^2)\cdot 2x \cdot \tan(\sqrt{x^2+1}) + \cos(x^2)\sec^2(\sqrt{x+1})\ \frac{1}{2}(x+1)^{-\frac{1}{2}}$

2. $f(x) = \sin x$. What is $f^{(2016)}(x)$, the 2016th derivative of f?

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

...

2016 divided by 4, the remander is 0.

$\Rightarrow f^{(2016)}(x) = \sin x$